

Phononic band gap with/without a defect layer in periodic and quasi-periodic structure

Arafa H^{*}. Aly, Ahmed Mehaney, Shrouk Eid

Physics Department, Faculty of Science, Beni-Suef University, Egypt

*) Email: arafa.hussien@science.bsu.edu.eg, arafa16@yahoo.com

Received 9/3/2017 Received in 21/4/2017 Accepted 21/6/2017

In the present paper, we focus on the ability of phononic crystals to treat the different elastic wave's frequencies at the same structure. Changing the periodicity of the structure from perfect and defect structure can modify the width/position of phononic band gaps. By comparing between one, two and four defect layers inside perfect ones, interesting results concerning the localized peaks with the band gaps were obtained. Since the width of band gaps and the number of localized peaks is increased by increasing the defect layers. Moreover, we pay more attention in comparing the band gaps between periodic and quasi-periodic structures. A multifunctional PnC structure could be obtained depend on periodicity and materials types were investigated and discussed. These results may be of potential importance in many engineering applications such as mechanical filters and vibration isolation devices.

Keywords: Phononic band gap; Defects; Crystal.

1. INTRODUCTION

Phononic Crystals (PnCs) are periodic composites structures can produce exceptional control over the propagation of elastic and acoustic waves [1-6]. The term "phononic" was derived from the name "phonon" which considered a quantization of lattice vibrations. PnCs was proposed in analog to photonic crystals, which have been introduced to control electromagnetic waves propagation [7-8]. PnCs can manipulate wavelengths from centimeters to nanometers ranges [9-10]. Therefore, PnCs were able to control, transmit and attenuate the propagation of sound, ultrasonic, hypersonic and heat frequencies. The intellectual property in PnCs is the existence of the so-called phononic band gaps. Within these band gaps, all frequencies are not allowed to propagate, while any frequencies outside band gaps are free to propagate. Based on these novel properties of PnCs, many different applications can be introduced in the field of acoustics and all elastic engineering applications such as filters, transducers, noise suppression and actuators [11-12].

The main principle factors in the formation of phononic band gaps are Bragg diffraction law and Brillouin zone [13-14]. When a mechanical wave incident on the face of a PnC structure, there are some parts of its energy will interact with the other parts at the interfaces between the constituent materials. As a result of the constructive interference with the Brillouin zone, waves are attenuated and band gaps formed. The other frequency bands known as pass bands, destructive interference dominated and waves effectively propagate through the structure. The width of the phononic band gap increases by using materials with a significant acoustic impedance mismatch.

The properties of phononic band gaps are considered a crucial factor in the formation of PnCs. There are several parameters have pronounced effect on band structure and band gaps behavior. Defects inside PnC structure can play a useful role on the width of band gaps and localized waves. Also, ordered and disordered composite structure can affect the properties of phononic band gaps [15-16].

In the present work, we paid more attention in studying the difference between periodic, quasi-periodic and defected PnCs structures on the propagation of elastic waves. Also, we emphases on the role of the defect layer in producing local resonance inside phononic band gaps. Defect layers introduce sharp peaks inside the phononic band gaps and increase band gaps width as well. The transfer matrix method (TMM) is used for this purpose. The Transmission coefficient is plotted for elastic waves; we will consider the transverse waves (S-wave). Moreover, we are interested in studying the propagation of waves through Fibonacci quasi- periodic PnC structure.

2. THEORETICAL TREATMENT

2.1. PnC Structure

The 1D PnCs $(A/B)^N$ structure is shown in Fig. 1. The structure consists of repeated unit cells. Each unit cell has two different materials A and B with the thinness, a_1 and a_2 , respectively. The number of periods is N . The lattice constant, $a = a_1 + a_2$.

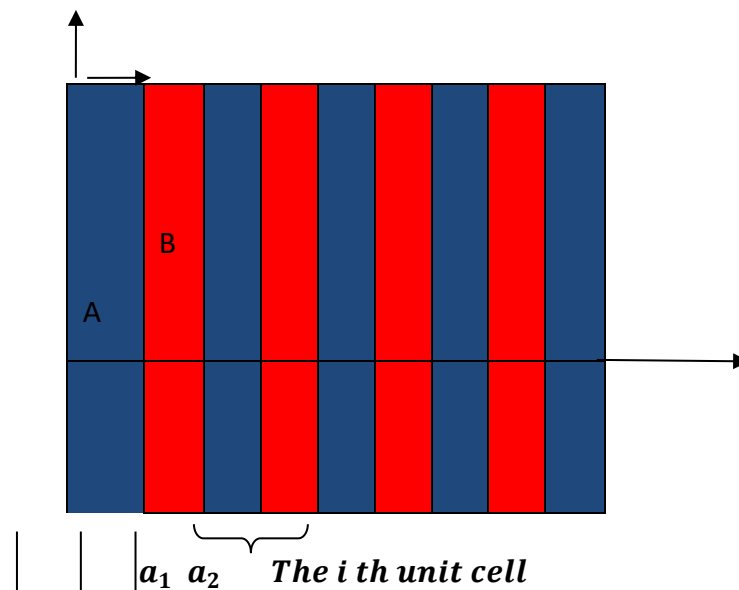


Figure.1. A schematic diagram of a perfect 1D PnC structure.

In order to mathematically describe how waves propagate through space and time, the partial differential equation for 1D wave propagation is defined as [5, 6].

$$\nabla^2 \varphi = c_L^{-2} \ddot{\varphi}, \quad \text{for longitudinal wave,} \quad (1)$$

$$\nabla^2 \varphi = c_T^{-2} \ddot{\varphi}, \quad \text{for transverse wave,} \quad (2)$$

where φ is the displacement potential, for the normal incidence case $v_x = \partial \varphi / \partial x$ is the displacement component, $c_L = \sqrt{(\lambda + 2\mu) / \rho}$ and $c_T = \sqrt{\mu / \rho}$ are the longitudinal and transverse waves speeds respectively, and $\nabla^2 = \partial^2 / \partial x^2$.

2.2. TMM and Transmission coefficient

The transfer matrix method of a 1D PnCs is discussed extensively in [5, 6]. The transmission coefficient of an elastic wave incident normal to the face of PnC structure subjected between two semi-infinite solid, is given by,

$$\frac{U_e}{U_0} = \frac{2E_0(T_{11}T_{22} - T_{12}T_{21})}{E_0(T_{11} - E_eT_{21}) - (T_{12} - E_eT_{22})}, \quad (3)$$

Where E_0 and E_e are Young's modulus of the two semi-infinite solids at the left and right of the PnC structure, U_1 and U_0 are the Transmitted and incident amplitude, respectively. $T_{ij} = T(i, j)$ are the elements of the total transfer matrix T : $T = T_n T_{n-1} \dots T_i \dots T_1$ with $T_i = T_2' T_1'$, and T_j' given in references [5, 7].

2.3. Defected 1D-periodic structure

In this subsection, we calculate the TMM of the defected PnCs structure considering that we have immersed a defect layer of thickness a_d to a periodic structure as shown in Fig.2.

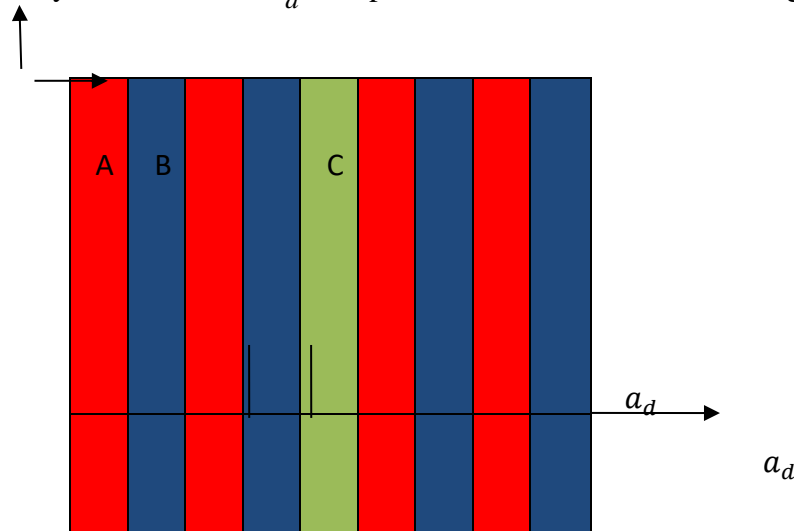


Figure.2. A Schematic diagram of a 1D defected PnC.

According to the previous description, our structure can be specified by the following matrix equation,

$$T = T_b T_d T_a. \quad (4)$$

Since T is the total transfer matrix of the structure and it is calculated as a product of three matrices, since the first one T_b describes the first periodic structure before the defect and it is given as

$$T_b = T_n T_{n-1} \dots T_i \dots T_1 \text{ with } T_i = T_2' T_1', \quad (5)$$

since n is the number of unit cells of the first periodic structure, T_d is the second matrix describes the defect layer and T_j' is given in references [5]. The third T_a describes the second periodic structure after the defect and it is given as

$$T_a = T_m T_{m-1} \dots T_i \dots T_1 \text{ with } T_i = T_2' T_1', \quad (6)$$

since m is the number of unit cells of the second periodic structure.

3. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the numerical results are presented to verify the comparison between phononic band gaps in periodic and Fibonacci quasi-periodic PnC structures. Also, the effects of single, pair and four defect layers on phononic band gaps are discussed. Table 1 present the materials used in our calculations.

Table 1. Materials constants

Materials	Mass density ρ (kg/ m ³)	Transverse wave speed V (m/sec)
Copper	8930	2300
Lead	11400	690
Epoxy	1180	1160
Gold	19320	1200
Nylon	1110	1048

3.1. Comparisons between phononic band gaps in periodic and Fibonacci quasi- periodic PnC structure

In this sub section, we will compare between the effect of periodicity (PnCs and quasi-periodic PnC structure) on the phononic band gaps by plotting the transmission coefficient for plane waves (transverse wave) incident normally on a 1D PnC. we consider the PnC structure consists

of (Al/epoxy)⁵ unit cells with thickness $a_A = a_B = 0.5 \times 10^{-6}$. Also, a Fibonacci sequence of quasi-periodic PnC structure of $n=13$ layers of the type (BABBABABBABBA).

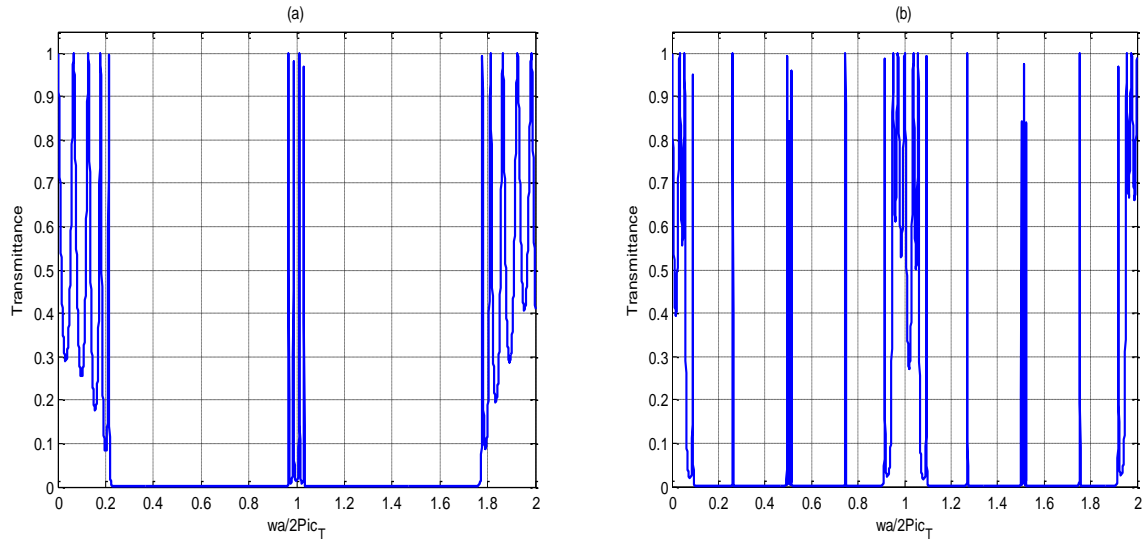


Figure.3. A plot of Transmittance and $\omega a/2\pi C_t$ (C_t is the sound transverse velocity in material B) for normal incident S- waves on: (a) 1D PnC consists (Al/epoxy)⁵, (b) A 1D Fibonacci sequence (BABBABABBABBA) quasi-periodic PnC consists of $n=13$ layers.

From Fig. 3(a) and (b) we note the great effect of changing the periodicity on the waves transmission in PnCs. We used copper and epoxy as consistent materials due to the large acoustic impedance mismatch between them. There are two different types of PnC structures, one is periodic 1D-PnCs and the other is the quasi-periodic structure (Fibonacci type). The formation of phononic band gaps was changed significantly in Fig. 3(b) than the periodic ones in Fig. 3(a). The band gaps are wider with little transmitted peaks in the case of periodic structure, while quasi- periodic structure increases the number of transmitted peaks with the band gaps. These results are due to the large change in the speed of elastic waves at many interfaces in the case of quasi-periodic structures than the periodic ones. Therefore, quasi-periodic structures could develop many transmitted windows through the phonic band gaps, which increase the opportunity of such structure in applications such as sonic filters.

3.2. Comparison between one, pair and four defect layers on phononic band gaps.

In this subsection, we will discuss and compare the effects of inserting a defect layer inside the previous structure on the phononic band gaps at three stages one, pair and four defect layers.

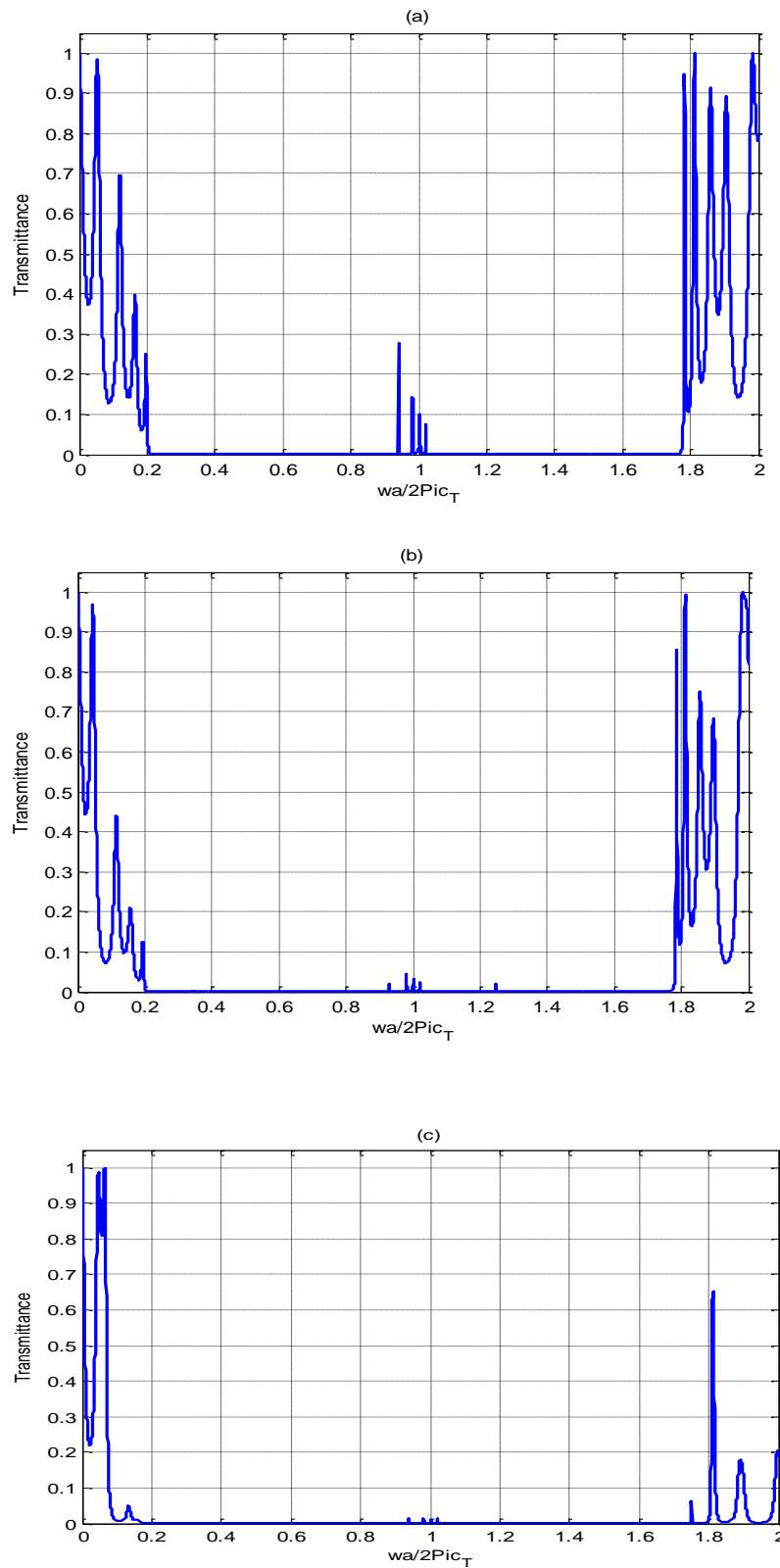


Figure.4. The transmittance of transverse waves through 1D- defected PnC structure, (a) nylon defect layer with $a_d = a_A$. (b) Two nylon defect layers with thickness $a_d = a_A$. (c) Two gold defect layers sandwiched by two nylon defect layers, each layer with thickness $a_d = a_A$. The defect layers in each case are inserted after material $j = 5$.

It is observed from Figs. 4(a), (b) and (c) that, the defect layer has a great effect on the phononic band gaps in all cases. In Fig.4 (a), there is one defect layer from nylon material inside the previous periodic structure. The large difference between nylon and constituent materials in elastic constants results in widening the phononic band gap and decreasing transmitted bands. Similarly, in Fig. 4(b) we considered two nylon defect layers inside the periodic ones. Consequently, the intensity of the transmitted peaks through the band gaps is decreased due to the previous reason. Moreover, in Fig.4(c), we considered the defect system consists of two hard materials (gold) sandwiched by two soft materials (nylon). Hence, the band gap increased greater than the last two cases due to the increment of mismatch between gold and nylon.

4. CONCLUSIONS

Based on the TMM, we have studied the effect of periodicity on the phononic band gaps at two studies:

- 1- Firstly, the difference between phononic band gaps in a periodic and Fibonacci quasi-periodic PnC structure. The quasi-periodic interrupt the periodicity of the periodic structure and induce transmitted peaks in the phononic band gaps.
- 2- Secondly, we performed a comparison between one, pair and four defect layers on the phononic band gaps. The four defect layers composed of two hard materials sandwiched by two soft materials result in the creation of very wide band gap.

Reference

- [1] A. H. Aly, A. Mehaney and M.F. Eissa, J. Appl. Phys. 118 (2015) 26
- [2] M. N. Armenise, C. E. Campanella, C. Ciminelli, F. D. Olio and V. M. N. Passaro, Phys. Procedia 3 (2010) 357
- [3] A. H. Aly, A. Mehaney and H. S. Hanafey, PIERS Proceedings 1043, (2012) 245
- [4] D. Manimekalai, Pradipkumar Dixit, Exp. Theo. NANOTECHNOLOGY 2 (2018) 45
- [5] A. H. Aly and A. Mehaney, Physica B 407 (2012)
- [6] J. Miklowitz, The Theory of Elastic Waves and Waveguides 22, North-Holland Publishing Company, New York, (1984)
- [7] A. H. Aly and D. Mohamed, J. Supercond. Nov. Magn. 18 (2015) 1699
- [8] A. H. Aly, S. A. El-Naggar and H.A. Elsayed, Optics Express 23 (2015) 15038
- [9] M. Babiker et al., J. Phys. C 18 (1985) 1269
- [10] A. H. Aly and A. Mehaney, Int. J. Thermophys. 36 (2015) 98
- [11] Y. Z. Wang et al., Arch. Appl. Mech. 80 (2010) 629
- [12] A. H. Aly, A. Mehaney and E. A. EL Rahman, int. J. Mod. Phys. B 27 (2013) 1350047
- [13] A. H. Aly and A. Mehaney, Indian J. Phys. DOI: 10.1007/s12648-017-0989-z (2017).
- [14] Yasser A. Abdel-Hadi, Exp. Theo. NANOTECHNOLOGY 2 (2018) 61
- [15] A. H. Aly and A. Mehaney, Chin. Phys. B. 25 (2016) 114301
- [16] C. Goffaux, and J.P. Vigneron, Phys. Rev. B: Condens. Matter 64 (2001) 075118

